MCR3U: Functions

Defining Equations of Functions

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Defining Equations of Functions

Besides being able to graph functions accurately, it is sometimes necessary to determine the equation of a function given its graph.

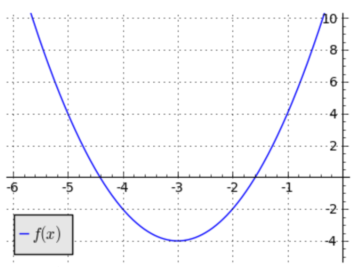
Being able to recognize the parent function is essential, as is identifying any transformations that have been applied.

Remember that a transformed function has the form $g(x) = af(b(x-c)) + d$, for real constants a, b, c and d.

Defining Equations of Functions

Example

For the function shown below, state the parent function, the transformations applied, its equation, and the domain and range.



Defining Equations of Functions

The graph is a parabola, so the parent function is $y = x^2$.

Since the vertex has shifted to $(-3, -4)$, there is a horizontal translation of 3 units left and a vertical translation of 4 units down.

For $y = x^2$, an integral point adjacent to the vertex would be located 1 unit to the right and 1 unit up.

For the transformed function, this point is located 1 unit to the right and 2 units up. Therefore, there is a vertical stretch by a factor of 2.

An equation is $f(x) = 2(x+3)^2 - 4$.

The domain is $\{x \in R\}$ and the range is $\{f(x) \in R \mid f(x) \geq -4\}$.

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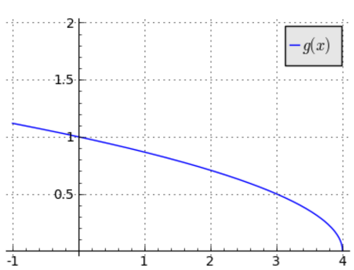
Example

For the function shown below, state the parent function, the transformations applied, its equation, and the domain and range.



Defining Equations of Functions

The parent function is $y = \sqrt{x}$.

The "origin" of the graph has shifted to $(4, 0)$, so there is a horizontal translation right of 4 units. There is also a horizontal reflection.

Examining adjacent integral points as before, the graph shows a vertical compression by a factor of $\frac{1}{2}$.

An equation is $g(x) = \frac{1}{2}\sqrt{-(x-4)}$, or $g(x) = \frac{1}{2}\sqrt{-x+4}$.

The domain is $\{x \in R \mid x \leq 4\}$ and the range is $\{g(x) \in R \mid g(x) \geq 0\}$.

Defining Equations of Functions

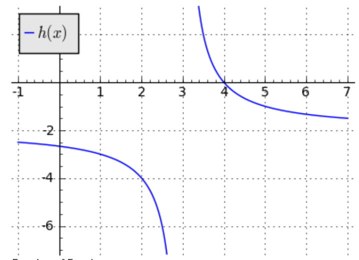
Sometimes it is just as important to watch key points that are *not* on a graph.

The locations of asymptotes and other discontinuities can indicate vertical and horizontal translations.

Defining Equations of Functions

Example

For the function shown below, state the parent function, the transformations applied, its equation, and the domain and range.



Defining Equations of Functions

The parent function is $y = \frac{1}{x}$.

The horizontal asymptote has shifted to $y = -2$, and the horizontal asymptote to $x = 3$. Thus, there is a vertical translation of 2 units down and a horizontal translation of 3 units right.

Examining adjacent integral points, the graph shows a vertical stretch by a factor of 2.

An equation is $h(x) = \frac{2}{x-3} - 2$.

The domain is $\{x \in R \mid x \neq 3\}$ and the range is $\{h(x) \in R \mid h(x) \neq -2\}$.

Questions?