MCR3U: Functions

Factoring Review (Part 1)

Common Factoring and Simple Trinomials

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Multiplying Polynomials By Monomials

Recall that we can use the Distributive Property of Multiplication to multiply a polynomial by a monomial.

$$\begin{aligned} 3x(4x^2 + 2x + 5) &= 3x \cdot 4x^2 + 3x \cdot 2x + 3x \cdot 5 \\ &= 12x^3 + 6x^2 + 15x \end{aligned}$$

When using the Distributive Property, each term of the polynomial is multiplied by the monomial.

Each term of the resulting polynomial contains the monomial as a *factor*.

Greatest Common Factors

The *Greatest Common Factor* (GCF) of two integers, a and b, is the largest integer that divides evenly into both a and b.

For example, the GCF of 24 and 20 is 4, while the GCF of 15 and 7 is 1.

We can extend the idea to include variables as well, by examining the exponents.

For example, the GCF of x^3y^5 and x^2y^9 is x^2y^5. In this case, we are limited by the smaller exponent of each variable.

Common Factoring

Common factoring is the process of extracting the GCF from all terms of a polynomial.

It is like the Distributive Property in reverse.

When factoring a polynomial, looking for a common factor should *always* be the first step, as it can greatly simplify the polynomial.

Common Factoring

Example

Factor $6x - 14$.

The GCF of $6x$ and -14 is 2.

Thus, $6x - 14 = 2(3x - 7)$.

We can check our work using the Distributive Property.

$$\begin{aligned} 2(3x - 7) &= 2 \cdot 3x - 2 \cdot 7 \\ &= 6x - 14 \end{aligned}$$

Note that the 2 is not thrown away! Common factoring simply allows us to *rewrite* the polynomial in a different form.

Common Factoring

Example

Factor $32x^4 - 12x^2 + 28x$.

When common factoring, we must find the GCF of *all* terms in the polynomial.

The GCF of $32x^4$, $-12x^2$ and $28x$ is $4x$.

Thus, $32x^4 - 12x^2 + 28x = 4x(8x^3 - 3x + 7)$.

Common Factoring

Your Turn

Factor $15a^3b^7 + 40a^3b^5$.

The GCF of $15a^3b^7$ and $40a^3b^5$ is $5a^3b^5$.

Thus, $15a^3b^7 + 40a^3b^5 = 5a^3b^5(3b^2 + 8)$.

Note that it was possible to factor out a^3 completely, whereas we could only factor out b^5 due to the second term.

Multiplying Two Binomials

Recall that when multiplying two binomials using the Distributive Property, there are four multiplications.

$$\begin{aligned}(x+2)(x+5) &= x\cdot x + 5\cdot x + 2\cdot x + 2\cdot 5\\ &= x^2 + 5x + 2x + 10\\ &= x^2 + 7x + 10\end{aligned}$$

The constant term of the resulting *trinomial* (10) is the product of the constants of the two binomials (2 and 5), while the coefficient of the central term (7) is the sum of these two values.

Factoring Simple Trinomials

A trinomial with a leading coefficient of 1 is a *simple trinomial*.

To factor a simple trinomial, we need to determine two constants that have a product equal to the constant term and a sum equal to the coefficient of the central term.

Factoring a Simple Trinomial

Given the trinomial $x^2 + bx + c$, if there exist two constants r and s such that $r \times s = c$ and $r + s = b$, then $x^2 + bx + c = (x + r)(x + s)$.

Factoring Simple Trinomials

Example

Factor $x^2 + 9x + 18$.

Two integers that have a product of 18 and a sum of 9 are 3 and 6.

Therefore, $x^2 + 9x + 18 = (x + 3)(x + 6)$

We can check our answer using the Distributive Property, as before.

$$\begin{aligned}(x+3)(x+6) &= x\cdot x + 6\cdot x + 3\cdot x + 3\cdot 6\\ &= x^2 + 9x + 18\end{aligned}$$

Factoring Simple Trinomials

Example

Factor $x^2 - 17x + 30$.

We want to find two integers that have a product of 30, and a sum of -17.

Since the product is positive, and the sum is negative, the two integers must both be negative.

Therefore, since $(-2)(-15) = 30$ and $(-2) + (-15) = -17$, $x^2 - 17x + 30 = (x - 2)(x - 15)$.

Factoring Simple Trinomials

Your Turn

Factor $x^2 - 14x - 15$.

We want to find two integers that have a product of -15, and a sum of -14.

Since the product is negative, the two integers must have different signs.

Since the sum is negative, the "larger" of the two integers must be negative.

The two integers are -15 and 1, and $x^2 - 14x - 15 = (x - 15)(x + 1)$.

Factoring Simple Trinomials

Your Turn

Factor $x^2 - 15x + 27$.

We want to find two integers that have a product of 27 and a sum of -15.

Since the product is positive, and the sum is negative, both integers must be negative.

All negative integer factors of 27 are -1, -3, -9, and -27. But $(-1) + (-27) = -28$ and $(-3) + (-9) = -12$.

Therefore, we say that the polynomial *does not factor* (DNF) over the integers.

Factoring Simple Trinomials

Example

Factor $x^2 + 2xy - 15y^2$.

Even though the terms contain two variables, we can still factor the trinomial using the previous technique.

Since the first term contains the variable x, there will be an x at the front of the brackets. Similarly, there will be a y at the end to produce the y^2 term.

We still want to find two integers (with different signs, the larger of which is positive) that have a product of -15, and a sum of 2.

The two integers are -3 and 5, so $x^2 + 2xy - 15y^2 = (x - 3y)(x + 5y)$.

Factoring Simple Trinomials

Example

Factor $x^4 + 9x^2 + 8$.

Even though the terms contain exponents greater than 2, we can still factor the trinomial using the previous technique.

We still want to find two integers (with same signs) that have a product of 8, and a sum of 9.

Since $x^4 = (x^2)^2$ we use x^2 for the variable in factored form. Notice how it appears in the central term.

The two integers are 1 and 8, so $x^4 + 9x^2 + 8 = (x^2 + 1)(x^2 + 8)$.

Factoring Simple Trinomials

Your Turn

Factor $x^6 + 5x^3y^4 - 24y^8$.

We still want to find two integers (with different signs, the larger of which is positive) that have a product of -24, and a sum of 5.

Since $x^6 = (x^3)^2$, and $y^8 = (y^4)^2$, we use x^3 and y^4 for the variables in factored form. Again, notice how they both appear in the central term.

The two integers are -3 and 8, so $x^6 + 5x^3y^4 - 24y^8 = (x^3 - 3y^4)(x^3 + 8y^4)$.

Questions?